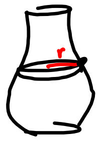
70 b) (0,6) integral wrt y



$$\int_0^6 \pi r^2\,dy = \int_0^6 \pi\left(\frac{c}{2\pi}\right)^2 dy$$

Need $c(y)$

$C = 2\pi r$

$r = \frac{c}{2\pi}$

$$\frac{1}{4\pi}\int_0^6 [c(y)]^2\,dy$$

$$\boxed{\frac{1}{2}\left(\frac{6-0}{12}\right)}\frac{1}{4\pi}\left[5.4^2+2(4.5^2)+2(4.4^2)+\cdots 6.3^2\right]$$

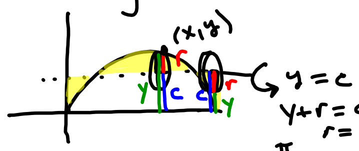
$\boxed{\frac{h}{2}}$

$$\frac{h}{2}\left[y_0+2y_1+2y_2\cdots\right] \qquad 208.255$$

$$\frac{\frac{b-a}{h}}{2} = \frac{\left(\frac{6-0}{12}\right)}{2} = \frac{1}{2}\left(\frac{6-0}{12}\right)$$

Dec 12-9:16 AM

69. $\int \pi r^2\,dx$



$y + r = c$

$r = c - y = c - \sin x$

$r + c = y$

$r = y - c$

$r = \sin x - c$

$$V = \int_0^\pi \pi(\sin x - c)^2\,dx$$

$$V = \frac{(2c^2\pi - 8c + \pi)\pi}{2}$$

$$\frac{dV}{dc} = 2(c\pi - 2)\pi = 0$$

min at $c = \frac{2}{\pi}$

$$\frac{d^2V}{dc^2} = 2\pi^2 > 0$$

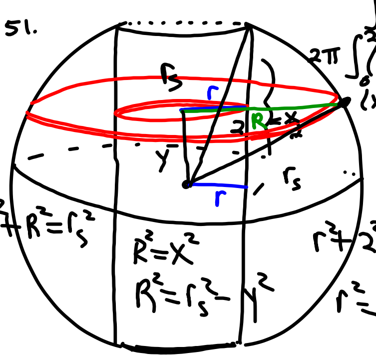
b) max. endpts.

$\boxed{C=0}$ $V = \frac{\pi^2}{2}$ max

$C = 1$ $V = \frac{(2\pi - 8 + \pi)\pi}{2}$

Dec 12-9:27 AM

51.



$$\int \pi R^2 - \pi r^2\,dy$$

$$2\pi\int_0^2 (r_s^2 - y^2) - (r_s^2 - 4)\,dy$$

$$2\pi\int_0^2 -y^2 + 4\,dy$$

$$\frac{32\pi}{3}$$

$y^2 + R^2 = r_s^2$

$R^2 = x^2$

$R^2 = r_s^2 - y^2$

$r^2 + 2^2 = r_s^2$

$r^2 = r_s^2 - 4$

Dec 12-9:41 AM

7.4 Length of a smooth curve

Approximate the length of the curve y = sin(x) from x = 0 to x = 2pi

$\Delta s^2 = \Delta x^2 + \Delta y^2$

$\Delta s = \sqrt{\Delta x^2 + \Delta y^2}$

$$s = \lim_{\Delta x \to 0}\sum\sqrt{\Delta x^2 + \Delta y^2}$$

$$= \lim\sum\sqrt{\left(1 + \frac{\Delta y^2}{\Delta x^2}\right)\Delta x^2}$$

$$= \lim\sum\sqrt{1 + \left(\frac{\Delta y}{\Delta x}\right)^2}\,\Delta x$$

$$\int_a^b f(x)\,dx = \lim\sum f(x)\Delta x$$

$$\lim_{\Delta x \to 0}\frac{\Delta y}{\Delta x} = f'(x)$$

$$\boxed{s = \int_a^b \sqrt{1 + [f'(x)]^2}\,dx}$$

$$\int_0^{2\pi}\sqrt{1 + \cos^2 x}\,dx$$

7.640

Dec 17-5:53 PM

Definition of arclength

$$\boxed{L=\int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx} \qquad y=f(x)$$

$$\boxed{L=\int_c^d \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy} \qquad x=g(y)$$

Dec 17-6:43 PM

Find the exact length of the curve $y=\frac{4\sqrt{2}}{3}x^{3/2}-1, 0\le x\le 1$

$$y'=\frac{3}{2}\cdot\frac{4}{3}\sqrt{2}\,x^{\frac{1}{2}}$$

$$y'=2\sqrt{2}\sqrt{x}$$

$$(y')^2=4\cdot 2\cdot x$$

$$(y')^2=8x$$

$$\int_0^1 \sqrt{1+8x}\,dx=\frac{13}{6}$$

Dec 17-6:49 PM

A vertical tangent

Find the length of the curve $y=\sqrt[3]{x}$ between (-8,-2) and (8,2) $\qquad x=y^3$

$$y'=\frac{1}{3}x^{-2/3}=\frac{1}{3\sqrt[3]{x^2}}$$

$$\int_{-8}^{8}\sqrt{1+\left(\frac{1}{3\sqrt[3]{x^2}}\right)^2}\,dx \overset{?}{=} 17.2606$$

$$\int_{-2}^{2}\sqrt{1+(3y^2)^2}\,dy=17.2607$$

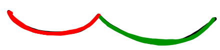
Dec 17-6:59 PM

A cusp

Find the length of the curve $y=x^2-4|x|-x$ from x=-4 to x=4

$x>0 \quad y=x^2-4x-x=x^2-5x$

$x<0 \quad y=x^2-4(-x)-x=x^2+3x$



$$\int_{-4}^{0}\sqrt{1+(2x+3)^2}\,dx+\int_0^4\sqrt{1+(2x-5)^2}\,dx$$

Dec 17-7:16 PM

Jan 3-12:44 PM